Math 62 12.7
Math 72 10.3 - 2nd

Nonlinear Inequalities and Systems of Nonlinear Inequalities

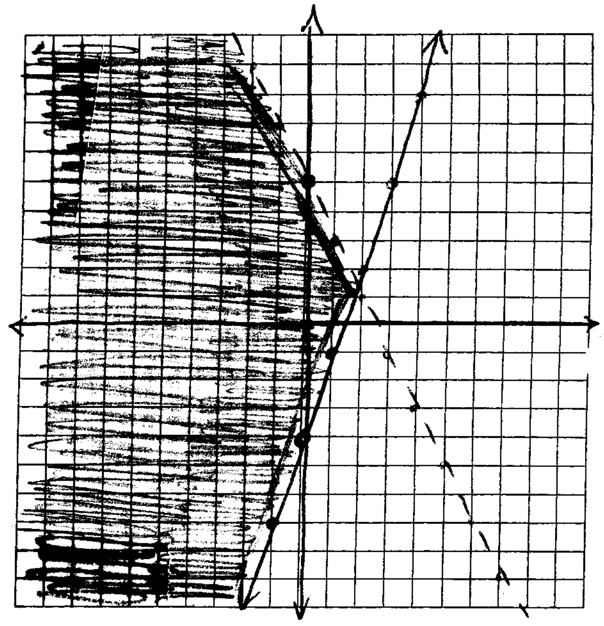
Math 70

Nonlinear Inequalities and Systems of Inequalities

Objectives
1) Graph a nonlinear inequality
2) Graph a system of nonlinear inequalities

Review from Math 45:

① Graph $\begin{cases} y < -2x + 5 \\ y \geq 3x - 4 \end{cases}$



$y < -2x + 5$

a line ⇒ dotted $<$ or $>$

slope -2, y-int 5

$y<$ shade downward ($y \leq$)

OR

Test a point with known location not on line

e.g (0,0)

$0 < -2(0) + 5$

$0 < 5$ true

Shade the side of $y = -2x + 5$ that contains (0,0)

$y \geq 3x - 4$

a line ⇒ solid $\geq$ or $\leq$

slope 3, yint -4

$y>$ shade upward ($y \geq$ also)

OR

Test a point with known location not on the line

e.g. (0,0)

$0 \geq 3(0) - 4$

$0 \geq -4$ true

Shade the side of $y = 3x - 4$ that contains (0,0).

(If test point is false, shade the other side of the line, not containing the test point.)

If the shaded areas do not overlap, the system of inequalities has no solution. (and no graph).

Math 70 MG 5/e 10.4

(2) Graph $\frac{x^2}{9} + \frac{y^2}{16} \geq 1$

Step 1: Change $\geq$ to $=$, identify conic, and graph.

If $\geq$ or $\leq$, use a solid line

(If $<$ or $>$, use a dotted (dashed) line.)

$\frac{x^2}{9} + \frac{y^2}{16} = 1$ is an ellipse

$\frac{(x-0)^2}{9} + \frac{(y-0)^2}{16} = 1$

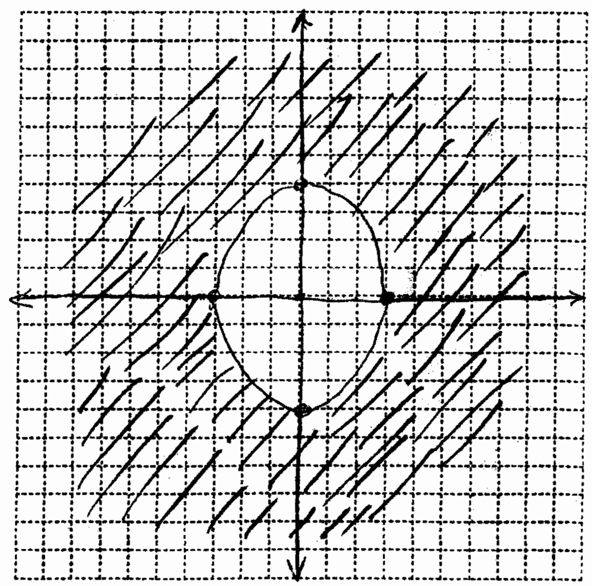
center $(0,0)$

x-direction 3

y-direction 4

Step 2: shade

Method 1: Use a test point. For circles, ellipses and hyperbolas, use the center.



$(0,0)$

$\frac{0^2}{9} + \frac{0^2}{16} \geq 1$

$0 + 0 \geq$

$0 \geq 1$ false

shade outside the ellipse

Method 2: For ellipses and circles

$\left.\begin{array}{l} ax^2 + by^2 > \# \\ ax^2 + by^2 \geq \# \end{array}\right\}$ greater than means outside

$\left.\begin{array}{l} ax^2 + by^2 < \# \\ ax^2 + by^2 \leq \# \end{array}\right\}$ less than means inside.

$\boxed{\text{To graph a system of nonlinear inequalities shade only the OVERLAP of shaded areas}}$

Math 70 MG 5/e 10.4

(3) Graph $\begin{cases} 4y^2 > x^2+16 & (A) \\ y \geq x^2 - 3 & (B) \end{cases}$

step 1: One inequality at a time, very neatly.

$4y^2 > x^2 + 16$

$>$ means dotted line; Arrange equation in standard form

Want x^2 and y^2 together

$4y^2 > x^2 + 16$

$-16 > x^2 - 4y^2$

$x^2 - 4y^2 < -16$ ← Note: Exchanging LHS and RHS changes $>$ to $<$

subtract x^2 and y^2 } hyperbola.

Need RHS 1.

$\dfrac{x^2}{-16} - \dfrac{4y^2}{-16} > \dfrac{-16}{-16}$ ← Note: Dividing by a negative changes $<$ to $>$

$\dfrac{-x^2}{16} + \dfrac{y^2}{4} > 1$ ← Negative at front of 1st term. Exchange order of 1st & 2nd terms

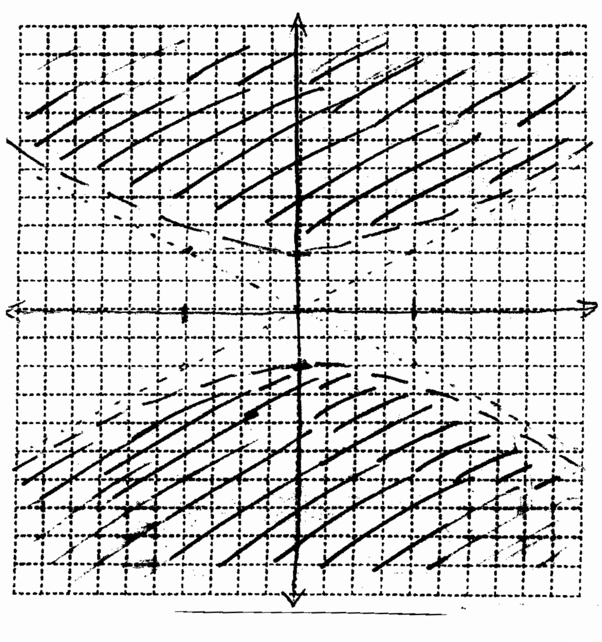
$\dfrac{y^2}{4} - \dfrac{x^2}{16} > 1$

center $(0,0)$ x-dir $\sqrt{16} = 4$ (under x^2)

y-dir $\sqrt{4} = 2$ (under y^2)

$y^2 - x^2$ opens up and down.

$>$ dotted line.

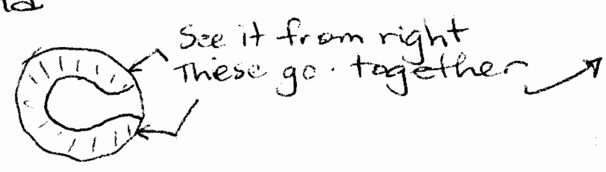


Step 2: Test center $(0,0)$

$\dfrac{0^2}{4} - \dfrac{0^2}{16} > 1$

$0 > 1$ false

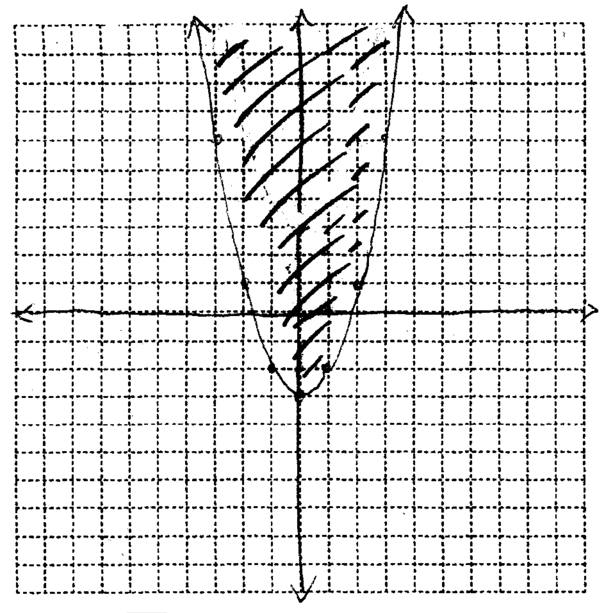
The shaded area of a hyperbola is like the fabric on a baseball; it "wraps around"



Math 70 MG 5/e 10.4

(3) con't $\begin{cases} 4y^2 > x^2+16 & (A) \\ y \geq x^2-3 & (B) \end{cases}$

But wait! We've only done the hyperbola. We have to add the other graph and determine where the shaded areas overlap.



step 3: Graph $y \geq x^2-3$

$y, x^2 \Rightarrow$ parabola up/down

$a=1$ opens up

$\geq$ solid line

vertex $(0,-3)$

Method 1: test a point not on the line (so do not use the vertex)

e.g. $(0,0)$

$0 \geq 0^2-3$

$0 \geq 3$ true

Method 2: For parabolas

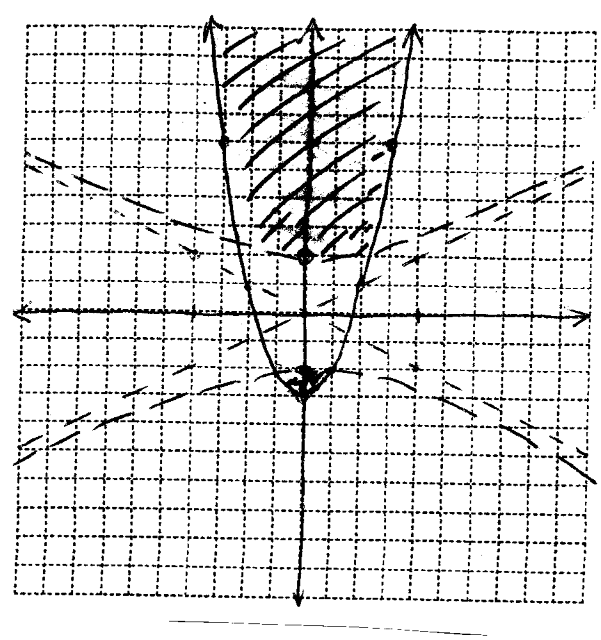
$y \geq$ or $y >$ shade upward from parabola

$y <$ or $y \leq$ shade downward from parabola

$x \geq$ or $x >$ shade rightward from parabola

$x <$ or $x \leq$ shade leftward from parabola

step 4: Shade the overlap



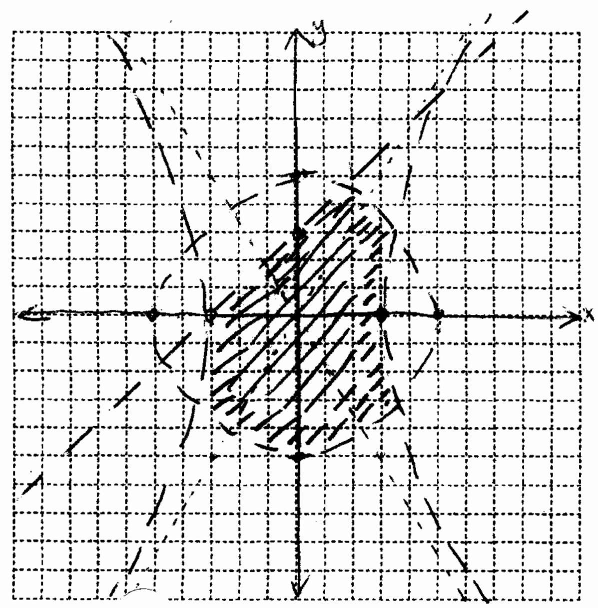
Math 70 MG 5/e 10.4

(4) Graph $\begin{cases} x^2 + y^2 < 25 & \text{(A)} \\ \dfrac{x^2}{9} - \dfrac{y^2}{25} < 1 & \text{(B)} \\ y < x + 3 & \text{(C)} \end{cases}$

step 1: Graph $x^2+y^2<25$ (A)
circle center $(0,0)$
radius $\sqrt{25}=5$
dotted line ($<$)
shade inward

step 2: Graph $\dfrac{x^2}{9} - \dfrac{y^2}{25} < 1$ (B)
hyperbola
center $(0,0)$
x-dir 3
y-dir 5
opens left & right
$x^2 - y^2$
dotted line $<$
shade between branches

step 3: Graph $y < x+3$ (C)
line
slope 1
y-int 3
dotted line $<$
shade downward $<$

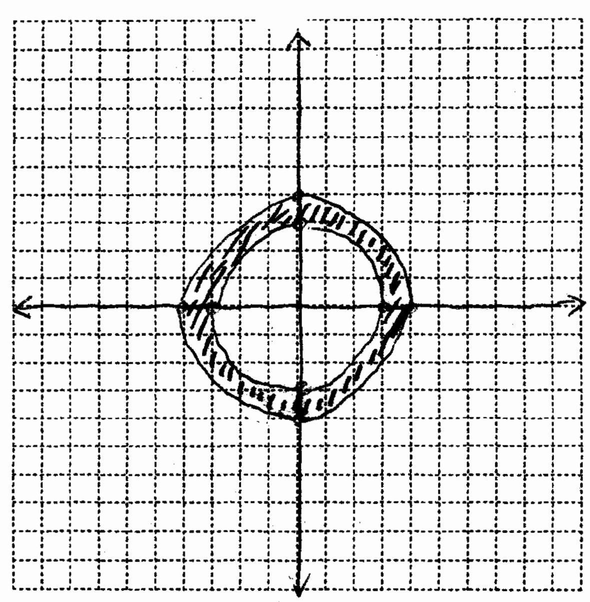


Extras ↓

(5) $\begin{cases} x^2 + y^2 \geq 9 & \text{(A)} \\ x^2 + y^2 \leq 16 & \text{(B)} \end{cases}$

(A) $x^2+y^2 \geq 9$
circle center $(0,0)$
radius $\sqrt{9}=3$
solid line
shade outward

(B) $x^2+y^2 \leq 16$
circle center $(0,0)$
radius $\sqrt{16}=4$
solid line
shade inward



Math 70 10.4

⑥ $\begin{cases} x^2 - y^2 \geq 1 & \text{(A)} \\ \dfrac{x^2}{16} + \dfrac{y^2}{4} \leq 1 & \text{(B)} \\ y \geq 1 & \text{(C)} \end{cases}$

(A) $x^2 - y^2 \geq 1$

hyperbola, solid

center $(0,0)$

$x^2 - y^2 \rightarrow$ left/right

x-dir 1

y dir 1

shade outward

(B) $\dfrac{x^2}{16} + \dfrac{y^2}{4} \leq 1$

ellipse, solid

center $(0,0)$

x-dir = 4

y-dir = 2

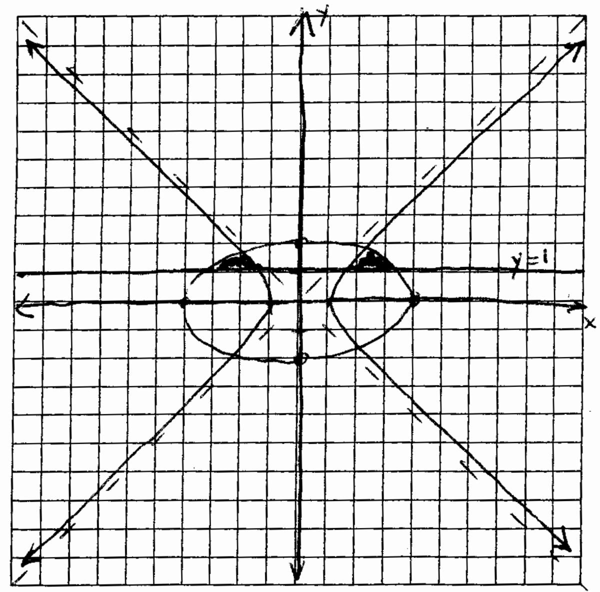
shade inward

c) $y \geq 1$

line, solid

horizontal

shade above



⑦ $\begin{cases} y^2 - x^2 \leq 1 & \text{(A)} \\ x \geq 0 & \text{(B)} \end{cases}$

(A) $y^2 - x^2 \leq 1$

hyperbola, solid

center $(0,0)$

$y^2 - x^2 \rightarrow$ up / down

x dir 1

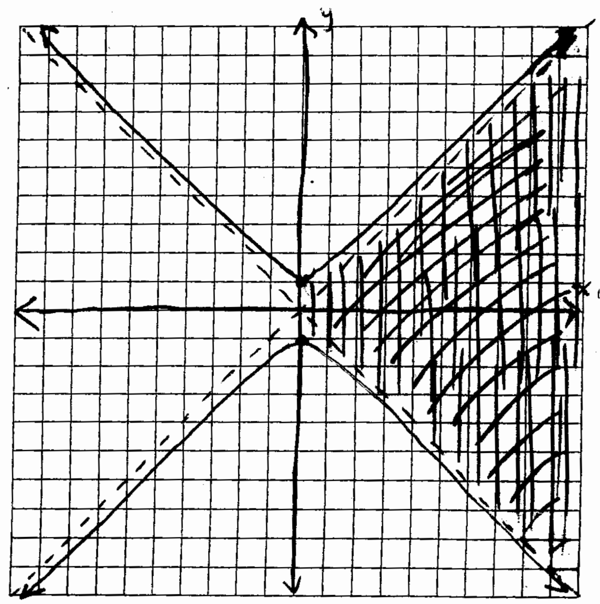
y dir 1

shade inward

(B) vertical line

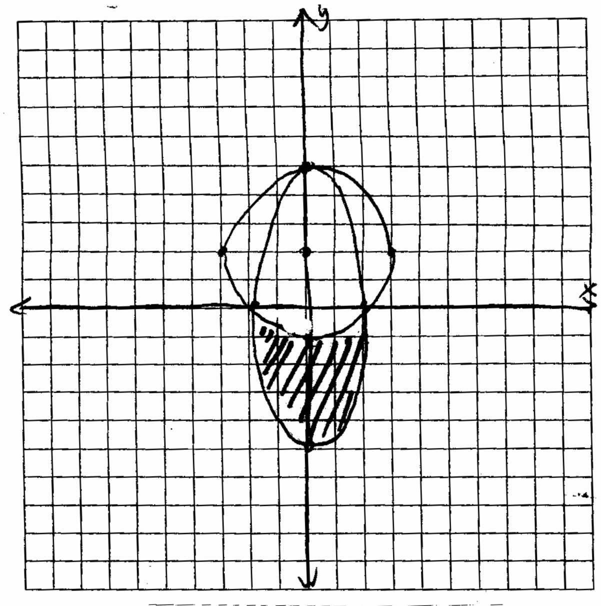
$x \geq 0$ solid

shade right



Math 70 10.4

8 $\begin{cases} x^2 + (y-2)^2 \geq 9 & \text{(A)} \\ \dfrac{x^2}{4} + \dfrac{y^2}{25} < 1 & \text{(B)} \end{cases}$



(A) $x^2 + (y-2)^2 \geq 9$

circle center $(0,2)$

solid

shade out

radius 3

(B) $\dfrac{x^2}{4} + \dfrac{y^2}{25} < 1$

ellipse center $(0,0)$

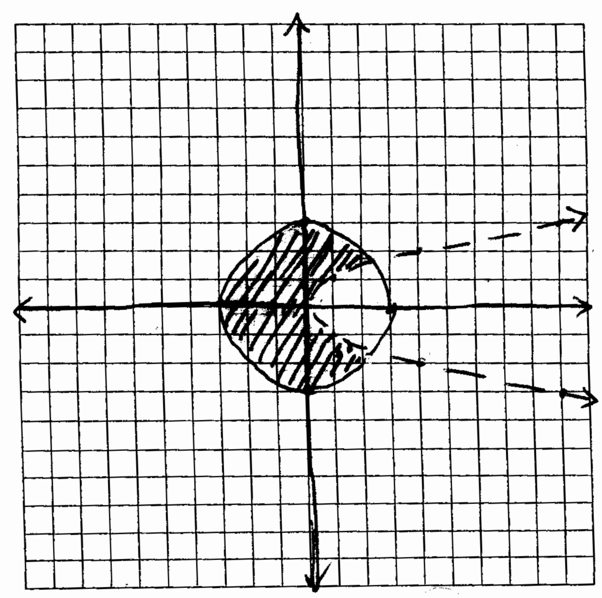
dotted

shade in

x-dir 2

y dir 5

9 $\begin{cases} x^2 + y^2 \leq 9 & \text{(A)} \\ x < y^2 & \text{(B)} \end{cases}$



(A) $x^2+y^2 \leq 9$

circle

center $(0,0)$

radius $\sqrt{9} = 3$

solid

Shade in

(B) $x < y^2$

parabola

vertex $(0,0)$

opens right

basic shape

dotted line

shade leftward

Math 70 10.4

(10) $\begin{cases} \frac{x^2}{4} + \frac{y^2}{25} \geq 1 & (A) \\ x < -y^2 - 2y + 6 & (B) \end{cases}$

(A) $\frac{x^2}{4} + \frac{y^2}{25} \geq 1$

ellipse
center (0,0)
x-dir 2
y-dir 5
solid border
shade out

(B) $x < -y^2 - 2y + 6$

paraboda
$x = y^2$ left/right
$a = -1$ left

$y = \frac{-b}{2a} = -\frac{(-2)}{2(-1)} = -1$

$x = -(-1)^2 - 2(-1) + 6 = 7$

dotted border
shade left